

Lecture 03: One-time Pad for Bit-strings

- We will see an encryption algorithm called “One-time Pad” for bit-strings
- In the future, we shall extend its domain to general abstract objects (for example, *groups*)

One-time Pad I

Yesterday.

- **Secret-key Generation:** Alice and Bob met and sampled a secret-key sk uniformly at random from the set $\{0, 1\}^n$, mathematically represented by $sk \sim \{0, 1\}^n$

Today.

- **Goal:** Alice wants to send a message $m \in \{0, 1\}^n$ to Bob over a public channel so that any eavesdropper cannot figure out the message m .
- **Encryption:** To achieve this goal, Alice computes a ciphertext c that encrypts the message m using the secret-key sk , mathematically represented by $c = \text{Enc}_{sk}(m) := m \oplus sk$. Here \oplus represents the bit-wise XOR of the bits of m and sk .
- **Communication:** Alice sends the cipher-text c to Bob over a public channel
- **Decryption:** Now, Bob wants to decrypt the cipher-text c to recover the message m . Mathematically, this step is represented by $m' = \text{Dec}_{sk}(c) := c \oplus sk$

- **Correctness:** Note that we will always have $m = m'$, i.e., Bob always correctly recovers the message
 - Note that in our case we always have $m = m'$
 - There are encryption schemes where with a small probability $m \neq m'$ is possible, i.e., the encryption scheme is incorrect with a small probability
- **Security:** Later in the course we shall see how to mathematically prove the following statement.

“An adversary who gets the ciphertext c obtains no *additional information* about the message m sent by Alice.”

One-time Pad III

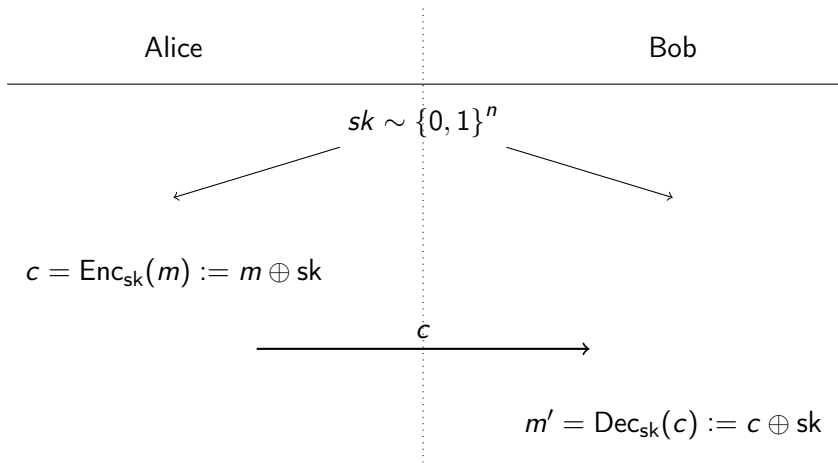


Figure: Pictorial Summary of the One-time Pad Encryption Scheme.

Dropping one Restriction makes the task Trivial

- Suppose we insist only on correctness and not on security
 - The trivial scheme where $\text{Enc}_{\text{sk}}(m) = m$, i.e., the encryption of any message m using any secret key sk is the message itself, satisfies correctness. However, this scheme is completely insecure!
- Suppose we insist only on security and not on correctness
 - The trivial scheme where $\text{Enc}_{\text{sk}}(m) = 0$, i.e., the encryption of any message m using any secret key sk is 0, satisfies the security constraint. However, Bob cannot correctly recover the original message m with certainty!
- So, the non-triviality is to simultaneously achieve correctness and security

- We are not trying to hide the fact that Alice sent a message to Bob
- We are trying to hide only the message that is being sent by Alice to Bob

Closing Remarks: Crucial Observation

- Fix a cipher-text c
- Consider any message m
- There exists a unique secret-key $sk_{m,c}$ such that
$$\text{Enc}_{sk_{m,sk}}(m) = c$$
- This observation shall be crucial to prove the security of the one-time pad private-key encryption scheme